Relational Operations: Overriding

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

Example: Calculate r overridden with {(a, 3), (c, 4)}

Hint: Decompose results to those in t's domain and those not in t's domain.

$$r \lhd t = \{(d, r') \mid (d, r') \in t \vee ((d, r') \in r \wedge d \notin dom(t))\}$$

r: relation; t: relation

Exercises: Algebraic Properties of Relational Operations

$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$

Define the image of set s on r in terms of other relational operations.

Hint: What range of value should be included?

Define r overridden with set t in terms of other relational operations.

Hint: To be in t's domain or not to be in t's domain?

Lab2: Relational Operators

Among persons, there is a special one: the celebrity. We define the *knows* relation which captures how the celebrity and other persons should be treated differently:

1. No person (including the celebrity) knows himself or herself.
2. The celebrity knows no one.
3. Everyone know the celebrity.

CONTEXT Celebrity_c0

CONSTANTS

k knows relation

c celebrity

P Set person

AXIOMS

axm1: $P \subseteq \mathbb{N}$

axm2: $c \in P$

axm3: $k \in (P \setminus \{c\}) \leftrightarrow P$

axm4: $k^{-1}[\{c\}] = P \setminus \{c\}$

axm5: $k \cap id = \varnothing$

END

MACHINE Celebrity_0

SEES Celebrity_c0

VARIABLES

r

INVARIANTS

inv1: $r \in P$

EVENTS

Initialisation

begin

act1: $r :\in P$

end

Event celebrity ⟨ordinary⟩ $\widehat{=}$

begin

act1: $r := c$

end

END

Functional Property

$$\text{isFunctional}(r) \Leftrightarrow$$
$$\forall\, s, t1, t2 \bullet$$
$$(\, s \in S \land t1 \in T \land t2 \in T \,)$$
$$\Rightarrow$$
$$(\, (s, t1) \in r \land (s, t2) \in r \Rightarrow t1 = t2 \,)$$

Q: Smallest relation satisfying the functional property.

Q: How to prove or disprove that a relation r is a function.

Q: Rewrite the functional property using contrapositive.